#### Eulerian Embroidery and Knitted Codes

Jane Butterfield

MathPath

2018







Portrait of Simon George by Hans Holbein the Younger, 1535



#### Portrait of Simon George by Hans Holbein the Younger, 1535









Elizabeth I. (From Thomasina Beck, The Embroiderer's Flowers)

#### Can we sew it?



#### Can we sew it?



#### Can we sew it?





Jane Butterfield (MathPath)











Could you similarly embroider a longer path?

Could you similarly embroider a longer path?



Could you similarly embroider a longer path?



 $P_{48}$ 




















































### $P_6$ with detours





#### Lemma

If you can embroider a pattern, then you can embroider it *starting* from any point.

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$$\sum_{i=1}^{n} \cdots \rightarrow \rightarrow \rightarrow$$

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Holden, J. (2007). The graph theory of blackwork embroidery. In S.M. Belcastro & C. Yackel (Eds.), Making Mathematics with Needlework: Ten Papers and Ten Projects.

Theorem

Any connected pattern can be blackwork embroidered.

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Theorem

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Proof idea:

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Proof idea: Delete some stitch in your pattern (say it's from hole x to hole y).

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#### Theorem

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### Proof idea:

Delete some stitch in your pattern (say it's from hole x to hole y).

If the resulting pattern is still connected, blackwork embroider it starting at hole x and ending at hole x. Then stitch x-to-y and y-to-x to complete your original pattern (one will be right side and one will be wrong side).

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Theorem

Any connected pattern can be blackwork embroidered.

Proof idea: Delete some stitch in your pattern (say it's from hole x to hole y).

If the resulting pattern isn't connected, then x is in one part and y is in the other. Embroider x's part (starting and ending at x), then stitch x-to-y, then embroider y's part, ending at y...

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If the resulting pattern isn't connected, then x is in one part and y is in the other. Embroider x's part (starting and ending at x), then stitch x-to-y, then embroider y's part, ending at y... Can we finish from there?

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Theorem

Any connected pattern can be blackwork embroidered.

Proof idea:

Delete some stitch in your pattern (say it's from hole x to hole y).

If the resulting pattern isn't connected, then x is in one part and y is in the other. Embroider x's part (starting and ending at x), then stitch x-to-y, then embroider y's part, ending at y... Can we finish from there?

Yes, because if we started on the right side in y's part we must end on the wrong side when we get back to y again.

## Blackwork patterns

The problem with blackwork "patterns" is that they don't tell you how to traverse them...



http://www.blackworkarchives.com

### A more helpful code

Row 2: P4, K1, P4 Row 3: K9 Row 4: P4, K1, P4 Row 5: K4, P1, K4 Row 6: P5, K1, P3 Row 7: K2, P1, K6 Row 8: P2, K1, P3, K1, P2 Row 9: K3, P3, K3 Row 10: P9 What do you think this is encoding?

### There are two types of stitch: **Knit** and **Purl**.

### Knit and Purl

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### Knitting code

Help me draw this on the whiteboard! Row 2: P4, K1, P4 Row 3: K9 Row 4: P4, K1, P4 Row 5: K4, P1, K4 Row 6: P5, K1, P3 Row 7: K2, P1, K6 Row 8: P2, K1, P3, K1, P2 Row 9: K3, P3, K3 Row 10: P9

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Here's the thing:

When you finish one row you start the next one from the nearest stitch...
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Row 3: K9

Row 4: P4, K1, P4

Row 5: K4, P1, K4

Row 6: P5, K1, P3

Row 7: K2, P1, K6

Row 8: P2, K1, P3, K1, P2

Row 9: K3, P3, K3

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A Purl is just a Knit from behind...

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Row 9: K3, P3, K3

Here's the thing:

We actually knit up from the bottom row.

#### Here's the thing:

#### We actually knit up from the bottom row.



Help me draw this on the whiteboard!(Correctly, this time!) Row 2: P4, K1, P4

Row 3: K9

Row 4: P4, K1, P4

Row 5: K4, P1, K4

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#### What's this a picture of?

#### What's this a picture of?



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What's this a picture of?





#### Knitting in the round

# Knitting in the round



We can increase the number of stitches on a row by adding in new ones as we go. Knowing that...

• How can I knit a triangle?

We can increase the number of stitches on a row by adding in new ones as we go. Knowing that...

- How can I knit a triangle?
- What would I make if I doubled the number of stitches on each row?

We can increase the number of stitches on a row by adding in new ones as we go. Knowing that...

- How can I knit a triangle?
- What would I make if I doubled the number of stitches on each row?
- How could you knit a circle?

We can increase the number of stitches on a row by adding in new ones as we go. Knowing that...

- How can I knit a triangle?
- What would I make if I doubled the number of stitches on each row?
- How could you knit a circle?

We can also "cast on" in a special way that allows us to connect the first and last rows together. Knowing that...

• How can I knit a torus?

We can increase the number of stitches on a row by adding in new ones as we go. Knowing that...

- How can I knit a triangle?
- What would I make if I doubled the number of stitches on each row?
- How could you knit a circle?

- How can I knit a torus?
- Could I knit a Klein bottle?



#### dr. sarah-marie belcastro http://www.toroidalsnark.net/mkkb.html